

- **[1]({DBF 1}))** Choose the CORRECT statement.
	- (A) There exists a ring R without identity, but a subring of R has identity
	- (B) There is a field F with an ideal I such that I is proper subset of F and  $I \neq \{0\}$
	- (C) Every finite commutative ring with identity is an integral domain
	- (D) There is a subring of  $\mathbb Z$  which is not an ideal of  $\mathbb Z$ , where  $\mathbb Z$  is the ring of integers under usual addition and multiplication.
- **2)** If  $n > 1$  and  $\gamma$  is the circle centered at *a* with radius *r*, then the value of the integral  $\int \frac{ac}{(z-a)^n}$ *dz*  $\int_{\gamma} \frac{a_2}{(z-a)^n}$  is (A)  $2\pi i$  (B)  $2\pi n$ (C)  $1$  (D) 0
- **3)** One endpoint of a diameter of a sphere is (2, 1, 0). If the center of the sphere is  $(3, 0, -2)$ , then the other endpoint of that diameter is
	- (A)  $(5, 1, -2)$  (B)  $(4, -1, -4)$ (C)  $(1, -1, -2)$  (D)  $\left(\frac{5}{2}, \frac{1}{2}, -1\right)$  $2^2$  $\left(\frac{5}{2}, \frac{1}{2}, -1\right)$  $\left[\frac{1}{2}, \frac{1}{2}, -1\right]$
- **4**) For  $\alpha = (x_1, x_2)$ ,  $\beta = (y_1, y_2)$  in the vector space  $\mathbb{R}^2$ , define the inner product  $< \alpha$ ,  $\beta > as$

 $< \alpha, \beta > = x_1 y_1 - x_2 y_1 - x_1 y_1 + 4 x_2 y_2$ 

If the two vectors  $(c, d)$  and  $(-d, c)$  are orthogonal, then

(A)  $d = \pm c$  (B)  $d = c = 0$ (C)  $d = \frac{1}{2} \left( -3 \pm \sqrt{13} \right)$ 2  $d = \frac{1}{2} \left(-3 \pm \sqrt{13}\right) c$  (D)  $d = \frac{1}{2} \left(3 \pm \sqrt{13}\right)$ 2  $d = \frac{1}{2}(3 \pm \sqrt{13})c$ 

**MP-1178 [2]**

5) If 
$$
x_n = (a^n + b^n)^{\frac{1}{n}}
$$
, where  $a > 0$  and  $b > 0$ . Then  $\lim_{n \to \infty} x_n$  is  
\n(A)  $\infty$  \n(B) 1  
\n(C)  $\max\{a, b\}$  \n(D)  $\min\{a, b\}$ 

**6**) If  $f(x, y, z) = x - 3y^2 + z$ , then the line integral of *f* along the straight line segment joining  $(0, 0, 0)$  and  $(0, 1, 0)$ 

(A) 1 (B) 
$$
\frac{\sqrt{3}}{2}
$$

(C)  $\sqrt{3}$  (D) –1

**7**) If  $z = x + iy$  is a complex number, let  $u(z) = 3xy^2 - x^3$ . The harmonic conjugate  $v(z)$  for *u* in  $\mathbb C$  that obeys  $v(0) = 1$  is

- (A)  $v(z) = -3x^2y + y^3 + 1$
- (B)  $v(z) = -3x^2y + y^2 + 1$
- (C)  $v(z) = -3xy^2 + y^2 + 1$
- (D)  $v(z) = -3xy^2 + y^3 + 1$
- **8**) Let  $y_0, y_1, \dots, y_n$  denote the set of values of *y* and  $\Delta y_k = y_{k+1} y_k$  denote the forward difference. If  $\Delta^k = \Delta(\Delta^{k-1})$ , the  $\Delta^3 y_0$  is
	- (A)  $y_3 y_2 + y_1 y_0$
	- (B)  $3y_3 2y_2 + 2y_1 3y_0$
	- (C)  $y_3 3y_2 + 3y_1 y_0$
	- (D)  $y_3 3y_2 3y_1 + y_0$

**MP-1178 [3]**

**(P.T.O.)**

**9**) Choose the WRONG statement about any sequence  $\{a_n\}$  of positive numbers.

(A) If 
$$
\sum_{n=1}^{\infty} a_n
$$
 diverges then  $\sum_{n=1}^{\infty} \frac{a_n}{1 + a_n}$  must also diverge  
\n(B) If  $\sum_{n=1}^{\infty} \sqrt{a_n}$  converges, then  $\sum_{n=1}^{\infty} a_n$  must also converge  
\n(C) If  $\sum_{n=1}^{\infty} a_n$  converges, then  $\sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{n}$  must converge  
\n(D) If  $\sum_{n=1}^{\infty} \sqrt{a_n}$  converges, then  $\sum_{n=1}^{\infty} n^2 a_n$  must also converge

- **10)** Let  $(Q, +)$  denote the group of rational numbers under addition. If  $\mathbb{Z}$  is the group of integers under addition, then
	- (A) *<sup>Q</sup> Z* is finite
	- (B) Every element of  $\frac{Q}{Z}$ *Z* is of finite order
	- (C) *<sup>Q</sup>*  $\frac{z}{Z}$  is cyclic
	- (D) All the above

**11)** Choose the WRONG statement among the following :

- (A) If *f* is continuous on [ $a$ ,  $b$ ], then *f* is Riemann integrable
- (B) If *f* is a bounded positive function on [ $a$ ,  $b$ ], then *f* is Riemann integrable
- (C) If *f* is monotonic on [ $a$ ,  $b$ ] then *f* is Riemann integrable
- (D) If *f* Riemann integrable on [a, b], then | *f* | is also Riemann integrable

## **MP-1178 [4]**

- **12)** The remainder obtained when 639! (factorial of 639) is divided by 641 is
	- (A) 2 (B) 1
	- (C) 639 (D) 640
- **13)** Which of the following can be the direction cosines of a line?

(A) 
$$
\frac{1}{3}, \frac{1}{3}, \frac{1}{3}
$$
  
\n(B)  $\frac{1}{2}, 0, \frac{\sqrt{3}}{2}$   
\n(C)  $\frac{1}{\sqrt{2}}, \frac{3}{2}, 0$   
\n(D) All of the above

- **14)** Which of the following is a correct argument to tell that the group of real numbers under addition  $(\mathbb{R}, +)$  is not isomorphic to group of non-zero real numbers under multiplication  $(\mathbb{R}^*, \times)$ ?
	- (A)  $(\mathbb{R}^*, \times)$  has a non-trivial finite subgroup, while  $(\mathbb{R}, +)$  does not
	- (B) If  $\varphi : (\mathbb{R}^*, \times) \to (\mathbb{R}, +)$  is a homomorphism, then  $\varphi(1) = 0$  and  $\varphi(r) = \varphi(r \times 1)$  $= \varphi(r) \times \varphi(1) = 0$ . Hence  $\varphi$  is not one-one and cannot be isomorphism.
	- (C)  $(\mathbb{R}, +)$  is cyclic, while  $(\mathbb{R}^*, \times)$  is not
	- (D) None of the above
- **15**) The function  $f(z) = \overline{z}$  is differentiable
	- (A) at nowhere in the complex plane
	- (B) at everywhere in the complex plane
	- (C) only at the origin
	- (D) only in the lower half plane (i.e.  $Im(z) < 0$ )

**MP-1178 [5] (P.T.O.)**

**16**) What is the Laplace transform of  $f(t)$ , where if  $0 \le t \le 2$  $(t) = \begin{cases} 1 & \text{if } t > 2 \\ 2 & \text{if } t > 2 \end{cases}$  $t$  if  $0 \leq t$ *f t t*  $=\begin{cases} t & \text{if } 0 \leq t \leq \\ 2 & \text{if } t > 2 \end{cases}$ 

(A) 
$$
\frac{1-e^a}{s^2}
$$
 (B)  $\frac{1-e^{2a}}{s^2}$ 

(C) 
$$
\frac{1-e^a}{s}
$$
 (D)  $\frac{1-e^{2a}}{s}$ 

- **17**) Which of the following polynomials is irreducible over  $\mathbb{Z}_7$ ?
	- (A)  $x^2 5$ (B)  $x^2 + 5$
	- (C)  $x^{13} 2$  (D)  $x^{19} 5$
- **18**) If  $S_6$  denotes permutation group on set of 6 symbols, then the number of 2 cycles in  $S_6$  is
	- (A) 12 (B) 15
	- (C) 6 (D) 30
- **19)** Identify the WRONG statement among the following :
	- (A) Gradient is defined only for scalar valued function
	- (B) Curl of a function is a vector valued function
	- (C) Gradient of divergence of a vector valued function is a scalar valued function
	- (D) Divergence of gradient of a scalar valued function is again a scalar valued function

**20**) Let the matrix of a linear transformation  $T : \mathbb{R}^3 \to \mathbb{R}^3$  with respect to standard

bases be 
$$
\begin{pmatrix} 1 & 0 & 3 \\ 0 & 2 & 1 \\ 2 & 1 & -1 \end{pmatrix}
$$
. Then which of the following is true?

(A) Nullity of  $T = 0$  (B) Nullity of  $T = 1$ 

- (C) Rank of  $T = 2$  (D) Rank of  $T = 1$
- **21)** Let G be a cyclic group of order *n* > 2 with identity *e*. Then which of the following is TRUE about  $f(x) = x^d - e$ ?
	- (A)  $f(x)$  has at least one solution  $x \neq e$  whenever  $gcd(d, n) > 1$
	- (B)  $f(x)$  has exactly *d* solutions for all *d*,  $1 \le d \le n$
	- (C)  $f(x)$  has at least one solution  $x \neq e$  if  $gcd(d, n) = 1$
	- (D)  $f(x)$  has exactly  $\phi(d)$  solutions for all  $d, 1 \le d \le n$
- **22**) Let  $f : \mathbb{R} \to \mathbb{R}$  be a function on set of real numbers. Fix  $x_0 \in \mathbb{R}$  consider the statement "Given any  $e > 0$ , there exists  $a \delta > 0$  such that whenever  $|x - x_0| < \delta, |f(x) - f(x_0)| < e$ ."

Negation of the above statement is :

- (A) For any  $e > 0$  there exists a  $\delta > 0$  such that whenever  $|x x_0| < \delta$ ,  $|f(x) - f(x_0)| \ge e.$
- (B) There exists an  $e > 0$  such that for all  $\delta > 0$ , whenever  $|x x_0| < \delta$ ,  $|f(x) - f(x_0)| \ge e.$
- (C) There exists an  $e > 0$  such that for every  $\delta > 0$ , there exists an *x* with  $|x - x_0| < \delta$  and  $|f(x) - f(x_0)| \ge e$ .
- (D) There exists an  $e > 0$  and a  $\delta > 0$  such that for some *x*, with  $|x - x_0| < \delta$ , we have  $|f(x) - f(x_0)| \ge e$ .

**MP-1178 [7] (P.T.O.)**

**23**) Which of the following functions  $f : \mathbb{R} \to \mathbb{R}$  is increasing on  $\mathbb{R}$ ?

(A) 
$$
f(x) = \frac{x^3}{3} + \frac{x^2}{2} + x + 1
$$
  
\n(B)  $f(x) = \frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{8} + 1$   
\n(C)  $f(x) = \frac{x^3}{3} + \frac{x^2}{2} - x + 1$   
\n(D)  $f(x) = \frac{x^3}{3} + \frac{x^2}{2} - \frac{x}{8} + 1$ 

- **24)** Which of the following two points lie on the same side of the plane  $2x + y + 5z - 4 = 0?$ 
	- (A)  $(1, 2, 1)$  and  $(1, 0, -1)$  (B)  $(3, -1, 2)$  and  $(-3, -1, 1)$
	- (C)  $(2, -4, 1)$  and  $(1, -2, 1)$  (D)  $(0, 1, 1)$  and  $(0, 1, -1)$

## **25)** The value of 8 18 0  $(3 j + 2)$ *j j*  $\sum_{j=0}$  (3*j* + 2)<sup>18</sup> (mod 27) is (A) 0 (B) 3 (C) 18 (D) 9

**26**) Let  $f(x)$  be bounded function on [0, 1]. Define a sequence  $\{a_n\}$  by 1  $\boldsymbol{0}$  $a_n = \int x^n f(x) dx$ . Then the value of  $\lim_{n \to \infty} a_n$  is (A) 1 (B)  $\infty$  $(C)$  0 1 2

**MP-1178 [8]**

27) If 
$$
f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ \frac{1}{n} & \text{if } x \text{ is rational} \end{cases}
$$
 is a function defined on  $[-a, a]$   $(a > 0)$ , then  
\n(A)  $U(P, f) = \frac{a^2}{2}$  and  $L(P, f) = 0$   
\n(B)  $U(P, f) = \frac{a^2}{2}$  and  $L(P, f) = \frac{-a^2}{2}$   
\n(C)  $U(P, f) = a$  and  $L(P, f) = -a$   
\n(D)  $U(P, f) = a$  and  $L(P, f) = 0$   
\n28) If  $x > 0$ , then

$$
\lim_{x \to 0^{+}} \int_{x}^{1} \log t \, dt
$$
\n(A) Diverges to  $-\infty$  \n(B) Converges to 0\n(C) Converges to  $-1$  \n(D) Converges to 1

**29**) Let  $J_n$  denote the set of all positive divisors of  $n > 1$ . Let  $*$  be the binary operation defined on  $J_n$  by  $a * b = LCM(a, b)$ . If  $e_n$  represents the identity element with respect to  $*$ , then

(A) 
$$
a * \frac{105}{a} = e_{105}
$$
 for all  $a \in J_{105}$   
\n(B)  $a * \frac{90}{a} = e_{90}$  for all  $a \in J_{90}$   
\n(C)  $a * \frac{135}{a} = e_{135}$  for all  $a \in J_{135}$   
\n(D)  $a * \frac{84}{a} = e_{84}$  for all  $a \in S_{84}$ 

- **30**) If R is a commutative ring with identity (with  $0 \neq 1$ ), then
	- (A) R must have at least one unit other than 1 and –1
	- (B) any element which is not a zero divisor must be a unit
	- (C) any zero divisor cannot be a unit
	- (D) set of all units need not form a group under the multiplication operation of ring

**MP-1178 [9] (P.T.O.)**

**31)** Which of the following CANNOT be the value (or values) of '*a*' if the following differential equation is to be of degree 2?

$$
-\left(\frac{d^2y}{dx^2}\right)^{3a} + \left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + y = 0
$$
\n(A) 0\n(B)  $\frac{1}{3}$ \n(C)  $\frac{2}{3}$ \n(D) None of the above

- **32)** Which of the following is NOT an exact differential equation?
	- (A)  $2xy \, dx + (1 + x^2) \, dy = 0$
	- (B)  $xe^{xy} dx + ye^{xy} dy = 0$
	- (C) sin *x* cos *y* dx cos *y* sin *x* dy = 0
	- (D)  $(3x^4y^2 x^2)dy + (4x^3y^3 2xy) dx = 0$
- **33**) For the function  $f(x) = x^3 2x 5$ , if the root lies in [2, 3], then the first approximation  $(x_1)$  of the root is

(A) 
$$
\frac{17}{35}
$$
 (B)  $\frac{35}{17}$   
(C)  $\frac{28}{15}$  (D)  $\frac{15}{28}$ 

**34**) Let the cross ratio of four points  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$  be m. If  $w_1$ ,  $w_2$ ,  $w_3$  and  $w_4$ are the images of  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$  respectively under the map 2 3 *z*  $w = \frac{2\lambda}{2}$ , then the cross ratio of  $w_1$ ,  $w_2$ ,  $w_3$  and  $w_4$  is

(A) 
$$
\frac{2m}{3}
$$
 (B)  $m^{\frac{2}{3}}$ 

(C) m (D) 
$$
\frac{2}{3m}
$$

**MP-1178 [10]**

- **35**) Which of the following is NOT a linear transformation from  $\mathbb{R}^3 \to \mathbb{R}^3$ ?
	- (A)  $f(x, y, z) = (2x, x + y, 3z)$
	- (B)  $f(x, y, z) = (x + y + 1, 2x + y, 3z + 2y)$
	- (C) *f* (*x*, *y*, *z*) = (2*x* + *z*, 0, *y*)
	- (D)  $f(x, y, z) = (0, 0, x + y + z)$
- **36**) Let V be a two dimensional vector space of *n* elements. Let  $v \in V$  be a nonzero vector. Then the number of linear transformations *f* from V to V such that  $f(v) = v$  is
	- (A) *n* (B) 1
	- (C) 0 (D)  $n-1$

**37**) Which of the following is a solution of the differential equation  $\frac{dy}{dx} + \frac{1}{2}xy = 3$  $\frac{dy}{dx} + \frac{1}{2}xy = 3x$  $+\frac{1}{2}xy = 3x$ .

(A)  $y = 6e^{-x^2/4} + c$ 

(B) 
$$
y = 6 + ce^{x^2/4}
$$

(C) 
$$
y = 6e^{x^2/4} + c
$$

(D) 
$$
y = 6 + ce^{-x^2/4}
$$

- **38)** In which of the following rings, there exists a bijection between set of zero divisors and set of units?
	- (A) Ring of integers  $\mathbb Z$
	- (B) Ring of rational numbers Q
	- $(C)$   $\mathbb{Z}_{15}$
	- (D)  $\mathbb{Z}_{16}$

## **MP-1178 [11]**

**39)** Which of the following correctly expresses the area of region enclosed by the parabola  $y = x^2$  and the line  $y = x + 2$ ?

(A) 
$$
\int_{-1}^{2} \int_{\sqrt{y}}^{y-2} dxdy
$$
  
\n(B)  $\int_{1}^{4} \int_{-\sqrt{y}}^{\sqrt{y}} dxdy + \int_{0}^{1} \int_{y-2}^{\sqrt{y}} dxdy + \int_{1}^{4} \int_{y-2}^{\sqrt{y}} dxdy$   
\n(C)  $\int_{0}^{1} \int_{-\sqrt{y}}^{\sqrt{y}} dxdy + \int_{1}^{4} \int_{y-2}^{\sqrt{y}} dxdy$   
\n(D)  $\int_{1}^{4} \int_{-\sqrt{y}}^{\sqrt{y}} dxdy + \int_{1}^{4} \int_{x^{2}}^{x+2} dydx$ 

**40**) The radius of curvature of the curve  $y = 3x^2 + 2x - 5$  at  $x = 1$  is

(A) 
$$
\frac{(65)^{\frac{3}{2}}}{6}
$$
 (B)  $\frac{(65)^{\frac{2}{3}}}{6}$  (C) 1 (D) 0

**41**) If one of the roots of the equation  $x^3 - 5x^2 + \frac{17x}{2} - 5 = 0$ *x*  $x^3 - 5x^2 + \frac{17x}{2} - 5 = 0$  is 2, then the other 2 roots are  $\mathcal{A}^{\mathcal{A}}$ 

(A) 
$$
-\frac{3}{2} + \frac{i}{2}
$$
 and  $-\frac{3}{2} - \frac{i}{2}$   
\n(B)  $\frac{3}{2} + \frac{i}{2}$  and  $\frac{3}{2} - \frac{i}{2}$   
\n(C)  $-\frac{1}{2} + \frac{3i}{2}$  and  $-\frac{1}{2} - \frac{-3i}{2}$   
\n(D)  $\frac{1}{2} + \frac{i}{2}$  and  $\frac{3}{2} - \frac{i}{2}$ 

**42)** Which of the following is NOT a homogeneous function? (A)  $f(x, y, z) = 2x^5y^2$ *z* (B)  $f(x, y, z) = x^3 + 2y^2x + zx^2$ (C)  $f(x, y, z) = x \cos \left( \frac{y + x}{z + y} \right)$  (D)  $f(x) = e^{\frac{y}{x}} z^3 + 2$ 

**MP-1178 [12]**

- **43)** Let V be a finite dimensional vector space of dimension *n* over a field F of *q* elements. Then the number of elements in V is
	- (A) *nq* (B) *nq*
	- (C)  $n + q$  (D)  $q^n$

**44)** If A and B are countable sets, then which of the following is uncountable?

- $(A)$   $A \cup B$
- (B) Cartesian product of A with B
- $(C)$   $A \cap B$
- (D) Union of power sets of A and B

**45)** If the population of a country doubles in 50 years, in how many years the population would have exceeded 3 times of the initial, assuming that the rate of increase of population with time is proportional to the population at that time

expressed in terms of years. (Use the approximate value of  $\frac{\log_e 3}{\log_e 2}$  = 1.585  $\log_e 2$ *e e*  $= 1.585$ )



(C) 65 years (D) 75 years

**46**) The *n<sup>th</sup>* derivative of  $f(x) = \cos^2(x)$  is

(A) 
$$
f^{(n)}(x) = 2^n \cos\left(2x + n\frac{\pi}{2}\right)
$$
  
\n(B)  $f^{(n)}(x) = 2^{n-1} \cos\left(2x + n\frac{\pi}{2}\right)$   
\n(C)  $f^{(n)}(x) = 2^{n-1} \cos(2x + n\pi)$   
\n(D)  $f^{(n)}(x) = 2^n \cos(2x + n\pi)$ 

**47**) Let *f* be a differentiable function on  $\mathbb{R}$ . If *f* has exactly two distinct roots, then

- (A)  $f'(x)$  also should have exactly two distinct roots
- (B) *f'*(*x*) should have exactly one distinct roots
- (C) *f'*(*x*) need not have a root at all
- (D) there is such an *f*, with  $f'(x)$  having infinite zeroes

**MP-1178 [13]**

**(P.T.O.)**

**48**) If  $z = re^{i\theta}$  and  $f(z) = u(r, \theta) + iv(r, \theta)$ , then which of the following represents the Cauchy-Riemann equations in the polar form?

(A) 
$$
\frac{\partial u}{\partial r} = \frac{\partial v}{\partial \theta}
$$
 and  $\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r}$   
\n(B)  $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}$  and  $\frac{\partial u}{\partial \theta} = -\frac{1}{r} \frac{\partial v}{\partial r}$   
\n(C)  $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}$  and  $\frac{\partial u}{\partial \theta} = -r \frac{\partial v}{\partial r}$   
\n(D)  $\frac{\partial u}{\partial r} = r \frac{\partial v}{\partial \theta}$  and  $\frac{\partial u}{\partial \theta} = -\frac{1}{r} \frac{\partial v}{\partial r}$ 

**49**) Which of the following is TRUE about the function  $f(x, y) = e^x \sin y$  defined on  $\mathbb{R}^2$ ?

(A) 
$$
\left(\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y^2}\right)^2 = 0
$$
  
\n(B)  $\left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial x}\right)^2 = 0$   
\n(C)  $\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = 0$   
\n(D)  $\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)^2 = 0$ 

**50**) Let  $x^3 + ax^2 + bx + c$  be a polynomial of degree 3 with roots  $\alpha$ ,  $\beta$  and  $\gamma$  (none of them is zero) and  $a \neq 0$ ,  $b \neq 0$ ,  $c \neq 0$ . Then the value of  $1 \t1 \t1$  $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$  is

(A) 
$$
\frac{-a}{b}
$$
 \t\t (B)  $\frac{-b}{c}$    
 (C)  $\frac{-c}{b}$  \t\t (D)  $\frac{-b}{a}$ 

## $\mathcal{H} \mathcal{H} \mathcal{H}$

Rough Work

